# Some uses of elasticity in instrument design

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The paper presents the text of a discourse delivered at the 1962 Exhibition of The Institute of Physics and The Physical Society, which reviews anti-distortion instrument mountings, elastic movements for rotation and translation, movement magnifying and reducing devices, elastic averaging schemes, simple harmonic force and constant force systems, elastic energy stores, and a slow relaxation device for transmitting rapid displacements only.

## 1. Introduction

Elasticity in the wrong place can much impair the working of an instrument. In the right place, it can be exploited to obviate the troubles caused by friction or gravity, or to provide forces with simple harmonic or constant characteristics; it can be used to make an energy store, as in a clock spring, or an averaging device such as a Merton nut. In this discourse I shall survey some of these uses; such a survey cannot take in the whole field, but it may serve as an introduction to some of the possibilities and difficulties inherent in the exploitation of elasticity.

Every material is to some extent elastic, and many materials have been employed to make elastic members. These materials have been mainly solids, each of which has to some extent the drawbacks of change of elasticity with temperature, and of creep and hysteresis. In general, the materials that have been most useful are those which, for given dimensions or weights, are capable of giving large elastic restoring forces over a large working range of displacements. In this group we have various steels, some non-ferrous alloys like phosphor bronze and beryllium copper, and fused quartz; choice between them usually depends on a requirement for some other property such as electrical conductivity, or non-magnetic characteristic, or ease of machining. For many applications, the relatively high negative temperature coefficients of elasticity of these materials (ranging, for the rigidity modulus, from $1 \cdot 2 \times 10^{-4}$ per degc for quartz through $2 \cdot 6 \times 10^{-4}$ for steel to $3 \cdot 8 \times 10^{-4}$ for copper, for example) are unimportant. Where small temperature coefficients are required, as in chronometer springs, there are a few alloys such as Elinvar, Nivarox, Ni-span C and Ni-span D where the temperature coefficient can be made smaller, and even positive, by suitable heat treatment.

This discourse will be dealing almost exclusively with solid springs, but it may be worth remembering that both liquids and gases are elastic. They suffer from high temperature coefficients of elasticity, but they should be free from 'creep'.

## 2. Fixed geometry

The fact that all materials are subject to elastic deformation means that any instrument, no matter how substantially it is made, will be liable to change its dimensions slightly if it is subject to external or internal forces. These forces can arise in many ways: temperature changes may cause different expansions in the components of the instrument, or a tilting of the instrument may cause a different disposition of the weight forces, or the operator's hand may add an extra and variable force to those already acting on the instrument. The first application of elasticity that I wish to mention is therefore, rather paradoxically, to the reduction or avoidance of undesired elastic deformation in instruments.

It is useful here to remember some advice attributed to Rowland that you should imagine your apparatus to be made of jelly, and to think of its behaviour accordingly. I have reminded myself of this advice in the last year (Jones 1961b) when designing instruments such as microbarographs where small internal displacements (of perhaps $10^{-6}$ cm) have to be measured, and where the instrument has to be protected from the effects of much larger external distortions. The solution here is to mount the instrument components on a stiff platform, and to support or suspend this platform by three relatively pliant members, giving a combination of kinematic and elastic design. Figure 1 shows some possible
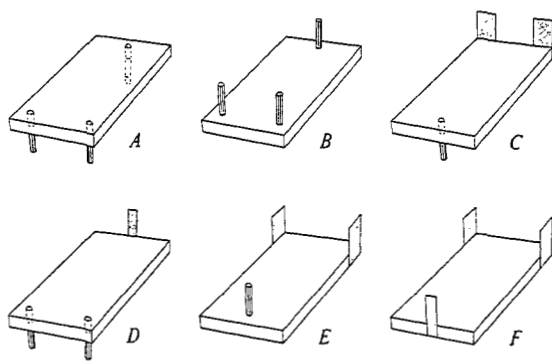


Figure 1. Some anti-distortion mountings for instrument platforms.

arrangements using wires or spring strips; it is possible by this simple device to reduce the effects of external distortions to one five-hundredth or so of their value, even though the pliant members are sufficiently stiff to give the instrument platform the 'feel' of being rigidly held. Spring (1958) has used a similar principle to hold optical components. Indeed, it is probable that Rowland (1902) used the same principle in supporting his ruling engine. This was supported on three slender legs (figure 2), giving an 'unengineered' look to the engine, but it is likely that these legs served to accommodate any change in dimensions of the supporting bench relative to the instrument.

The change of dimensions caused by the redistribution of weight forces occurs to some extent in any instrument that has to function in different vertical orientations, and this change is particularly acute in large telescopes where heavy mirrors are involved. Much ingenuity has gone into the

away to a reservoir and causing the tube to contract. In this way the hydraulic pressure is made to bring back the tube to its designed working length, whatever the external load. A suitable skeleton of these constant length members will simulate an ideally rigid body.
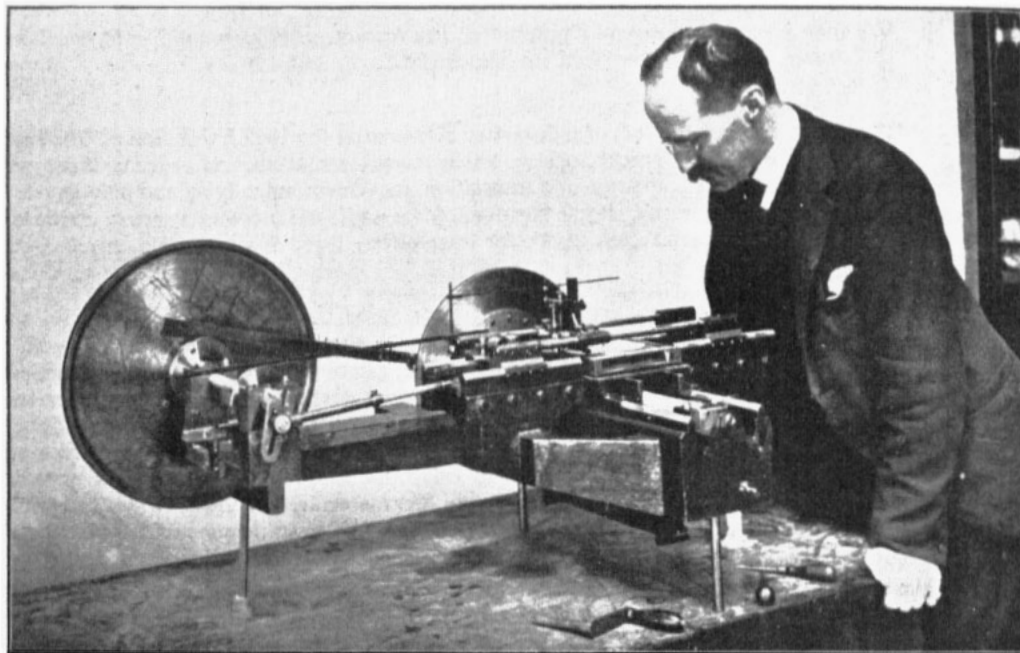


Figure 2. H. A. Rowland with his ruling engine for diffraction gratings.

devising of automatic supports for redistributing the thrusts on the mirror as its elevation is changed (Sisson 1960), and experiments are in progress under Professor Redman to see whether much thinner mirrors could be suitably compensated. Some day we may even see thin mirrors in which the figure is monitored optically, and appropriate elastic corrections applied by a battery of pneumatic bellows at the back of the mirror.

An elegant idea for the construction of a near-ideal rigid framework, suitable for structures such as large telescopes, is due to Dr. B. N. Wallis (1955), who exploits the phenomenon of elastic deformation to neutralize itself. To understand Dr. Wallis's idea, it is necessary only to remember the definition of a rigid body as one such that the distance between any two points in it is invariant. Dr. Wallis therefore constructs his framework from a number of members, each of which consists of a hollow tube, filled with oil under pressure; under no external load, the oil pressure causes the tube to lengthen elastically to its designed working length. Down the axis of the tube runs a rod (which could if necessary, be made of Invar), and there is a device for detecting any difference between the length of the tube, which is taking the external load, and the rod, which is not. The difference signal is made to change the oil pressure in the tube, in such a way as to bring its length back to that of the rod. The mechanism can consist simply of a piston attached to the rod, so that it opens a port to a high pressure supply if the outside tube is shortened by an external compression load, thus causing it to be pumped up to its original working length; if the tube is too long, the relative motion of piston and tube opens another port, allowing oil to leak

## 3. Variable geometry

Nearly all instruments have at least one part that moves relatively to the rest, and in many instances this motion is effected by sliding the one part over the other, easing the friction by careful choice of surface materials, or by lubrication, or by using ball, roller, or air bearings. As Clerk Maxwell (1876) pointed out, however:

"In another class of instruments sliding and rolling are entirely done away with, and sufficient freedom of motion is secured by the pliability of certain solid parts. . . . The motion of the piece is then affected by the elastic force of the suspension apparatus, but this force is much more regular in its action than friction, and its effects can be accurately taken account of, and a proper correction applied to the observed result."

The fundamental limitation to the employment of elastic linkages for controlled movement is that the range of useful deflection of an elastic member is fairly small, usually not larger than, say, 10% of the maximum dimension of the member. The opportunities for the employment of elastic linkages therefore lie in those instruments where the amount of motion required is substantially less than the outside dimensions of the instrument. Fortunately, many instruments fall into this category, and special forms of construction thus become possible. For example, the opening of the two jaws of a spectrometer slit is at first sight philosophically similar to the opening of the two doors of an underground train, and indeed the slit mechanism is often made on this principle. But there is in fact a difference that can be exploited.

The train doors have to open to a dimension which is comparable with the width of the train, while the slit jaws are always close together compared with the width of the spectrometer. Very satisfactory elastic linkages can thus be designed for moving the slit jaws, giving a better performance than a miniaturized 'engineering' or kinematic design, for a smaller effort of construction and without adding appreciably to the overall size of the spectrometer.

Most instrument motions require one degree of freedom only, so that the elastic linkage should be flexible in this one degree, and stiff in all the others. Historically, linkages permitting rotation preceded those for translation, and we shall therefore now look at the standard rotational linkages.

## 4. Rotation

The earliest linkages were the leaf spring and the torsion balance. Good bibliographies on these and other flexure devices are those by Geary (1954, 1960); another useful reference is N.P.L. Notes on Applied Science, No. 15 (National Physical Laboratory 1956).

The leaf spring hinge (figure 3A) was probably first used as a near-frictionless suspension for a clock pendulum. The
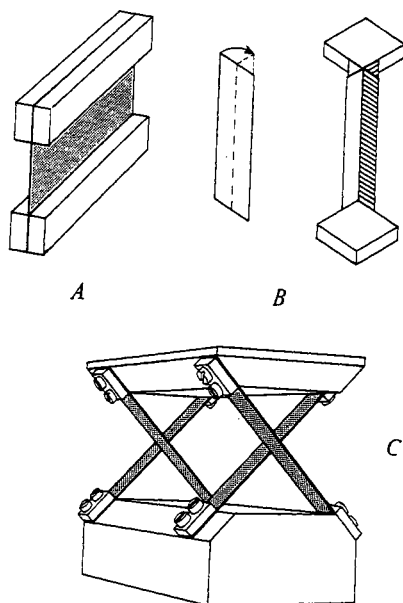


Figure 3. Spring hinges.
A, leaf spring; B, angle-spring; C, cross-strip.

effective axis of rotation varies with the displacement, but is approximately in the mid-point of the strip for small displacements, for a pure torque. The hinge has the defect that it is comparatively weak against twisting—to resist this, it should be made as wide as possible, and with a short free length. Its simplicity gives it value.

The torsion balance is one of the classical devices of experimental physics. It was invented by Michell about 1750 (Cavendish 1798), and also by Coulomb (1785), and it has since been in constant use, particularly when small torques have had to be measured. The original torsional members were wires or threads of circular cross section; Ohm (1826) appears to have been the first to realize that a strip would be

much weaker torsionally for the same cross section, and would thus be most suitable when mechanical or electrical loads have to be carried. Since World War II, shock-proof torsional suspensions have been increasingly used in electrical instruments to replace jewelled pivots. The absence of 'sticking' in a torsion balance makes it possible to observe reliably very small changes in torque, and to make meaningful observations of $10^{-9}$ radian change in orientation (Jones 1960). Typical of the very small torques that can be measured by a torsion balance is Holbourn's (1936) observation of the angular momentum of circularly polarized light; he was able to detect a torque of $2 \times 10^{-11}$ dyn cm.

The cross-strip hinge is a useful device, since it has considerable stiffness in all degrees of freedom other than rotation about the line of intersection of the planes of the strips. The double cross-strip (figure 3C) is particularly good in this respect. For an accurately located (within a few micrometres) axis of rotation, the hinge has to be well made, and rotation confined to a few degrees; the operating torque should be applied in the plane lying as symmetrically as possible between the strips, but owing to the great strength of the hinge, neither this condition nor the loading of the hinge is likely to be critical.

Various other kinds of elastic hinge are possible. Of these I have found the angle-spring (figure 3B) hinge the most useful, because it is so easily made, and takes up so little room. Details of its performance were given in this Journal (Jones 1955). A recent and impressive example of the use of flexure pivots is by Marsh (1961) in a micro-tensile testing machine for measuring the strengths of metal 'whiskers'.

## 5. Translation

For translation, the basic elastic linkage is the parallel-spring movement, pioneered by E. M. Eden at N.P.L.; it may well be an old device, but I know of no reference before Eden's work. It consists of two rectangular blocks separated by two parallel spring strips (figure 4A). If the lower block is fixed, and the upper block is pushed sideways, it will be deflected to a position approximately parallel to its original one. Indeed, if the blocks and springs are perfect, and the force is applied in a plane midway between the blocks, the parallelism of movement should be exact (figure 4C). Direct pushing on the top block will usually introduce only a trivial error. Some of the more important design factors are discussed in papers by myself and Young (1956).

Departures from parallel motion over displacements of a few millimetres may be reduced comparatively easily to the order of 10 seconds, for spring and block dimensions of a few centimetres, and with care to about 1 second. This has been done both at Aberdeen and at the National Physical Laboratory (1961), where a Michelson interferometer mirror for infra-red spectrometry has been moved over 10 mm displacement with a deviation not greater than $\pm 1$ second in either azimuth or elevation. Uniform spring material is necessary for such precision. If such small deviations are needed in, say, azimuth only, with less stringency in elevation, spring errors can often be compensated by stiffening the springs at appropriate points by small adjustable clamps (Jones 1951); it is in addition possible to bring errors simultaneously down to within $\pm 1$ second over several millimetres in both azimuth and elevation by the use of these clamps, but this may involve luck and insight.

An interesting recent application of the parallel-spring movement is that by Bradley (1962) to a scanning Fabry-

Perrot spectrometer with one plate oscillating at frequencies up to 1000 c/s.

While the parallel spring system can give parallel movement, the top block is constrained to move on a curved path, determined by the constant length of the strips. Any point
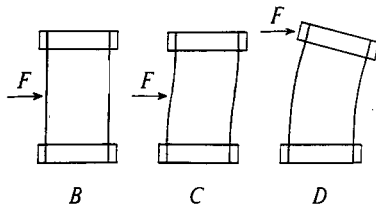


A



B        C        D
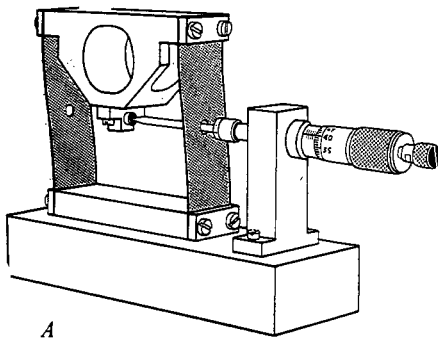
Figure 4. A, parallel spring movement; B and C, effect of pushing along correct line, giving purely parallel motion; D, component of cantilever bending which is directly superimposed on parallel motion by pushing moving platform; this is often unimportant.

on the top block thus follows a curve that is approximately a circle of radius equal to the length of the strips. The consequent slight decrease in the perpendicular distance between the blocks is immaterial for many purposes, and the simple parallel spring movement is thus capable of widespread application. When truly rectilinear motion is required, two simple movements can be combined, as in figure 5. Here block A and the compound top block FHG constitute one parallel motion with springs R and U; and blocks H and K form a parallel motion with springs S and T. All springs
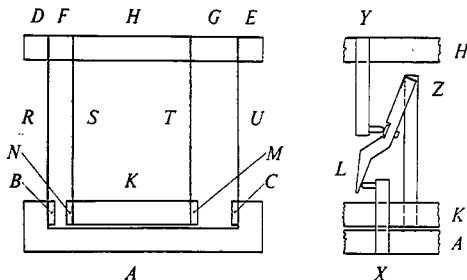


Figure 5. Left, rectilinear spring movement. Right, 'slaving' device for improving rectilinear motion when platforms are heavy.

are of the same dimensions, and so if block A is fixed, and block K is pushed sideways, K will move in a straight line since it must move parallel to H, which must move parallel to A, and the distance between H and K will be equal to that between H and the uppermost faces of A. Thus K moves at a constant height relative to A. The first use of this principle appears to have been by Clay (1937).

Various other arrangements are possible. Figure 6 shows a movement where the parallel spring units are mounted
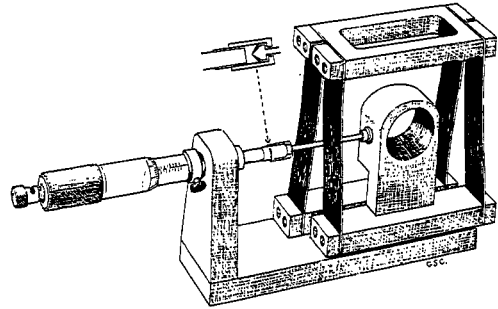


Figure 6. Side-by-side compound parallel spring arrangement for rectilinear cross motion on an optical bench.

side by side. This is a cross motion for an eyepiece or other component on an optical bench. The departure from rectilinearity is 70 $\mu$m at the extreme of 20 mm of travel, but is less than 10 $\mu$m over 15 mm. The moving platform stays parallel to the fixed one to within 10″ over 20 mm movement; this is less than the change caused, as the weight of the eyepiece is translated, by the bending of the pillar on which the whole unit is mounted on the optical bench saddle. This unit illustrates the ease with which satisfactory movements can be made, with less effort than is required for kinematic construction, and with automatic freedom from backlash.

The main difficulty with rectilinear spring movements arises from the effects of gravity. In figure 5, for example, the motion of K is rectilinear so long as both this platform and the intermediate platform FHG are weightless. The effect of weight, if the assembly is mounted with the springs upright, is to place springs S and T in tension, and R and U in compression. The effect of this is to bend R and U rather more, when the platform K is displaced, and S and T rather less than when the platforms are weightless. If the platform K is carrying an appreciable weight, the vertical distance between H and K is thus rather greater than that between H and A, and K therefore tends to drop as its displacement is increased, and its motion is no longer rectilinear.

Apart from keeping weight to a minimum, there are two ways of dealing with this trouble. The first is to mount the movement on its side, so that gravity no longer acts along the springs. This works well for small displacements, but the springs tend to twist at large displacements. For these latter, it is better to adopt a practice first suggested by Plainevaux (1954a, b), who pointed out that the correct geometry could be ensured by 'slaving' the intermediate platform FHG to move exactly one half as much as the operating platform K. This slaving can be achieved in various ways; a possible method, which I have not tried, would be by hydraulically operated sylphon bellows, since the 2 : 1 reduction in motion required could be achieved by appropriate choice of cross sections, and no friction need be

introduced. Figure 5 shows a simple lever mechanism for effecting the slaving, with which it proved possible to translate an 0·5 kg weight through 20 mm without departing by more than about 0·1 $\mu$m from a straight line (Jones and Young 1956). An additional 3 kg weight caused a departure of not more than 2 $\mu$m from the original line.

For smaller weights and movements, slaving is usually unnecessary, but it is worth pointing out that more attention must be paid to weight considerations in elastic design than in kinematic design; the extra consideration is generally well repaid by the freedom from friction and wear.

Improved precision can be gained by making symmetrical spring systems; two possible systems have previously been sketched (Jones 1951, Jones and Young 1956), and a third is illustrated in figure 7. This consists of two side-by-side
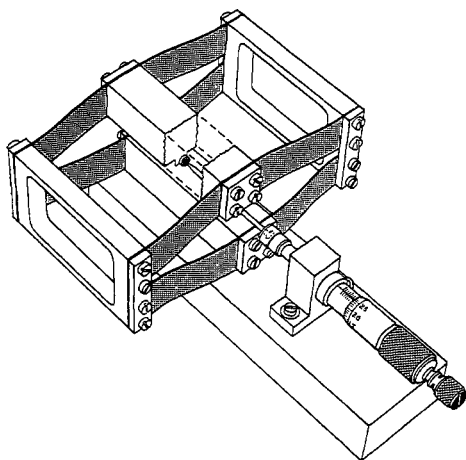


Figure 7. Symmetrical rectilinear movement.

movements of the type sketched in figure 6, and is simple to construct since, although in principle eight springs are used, each spring forms a continuous length with its balancing neighbour, and thus only four strips have to be cut and manipulated. The arrangement gives a relatively compact construction with a block/spring geometry favourable to good precision.

## 6. Lever mechanisms

It is often useful to have a frictionless mechanism for converting a large motion to a small one, or vice versa. There are several spring arrangements that can be used for these purposes.

If a strong spring and a weak spring are joined in series, and a displacement applied to the free end of the weak spring, while the other end of the strong spring is fixed, the joint between the springs moves by a fraction of the applied displacement dependent on the ratio of the strength of the springs. This makes a simple device for reduction of movement, but a little care is necessary in design. If each of the two springs is a leaf, then the movement of the joint may be quite complex (figure 8B) and non-linear. It is much better to make the strong component a parallel spring unit (figure 8C), and thus constrain the joint to a predetermined path. Numerous variants of this device are possible.

The twin-strip mechanism (figure 8A) was used by N.P.L.

in the Eden–Rolt comparator. The two strips are joined together at one end; at their other ends they are fixed to components one of which is usually itself fixed, and the other (the upper spring in the figure) constrained to move in a straight line by a parallel spring movement. The joined
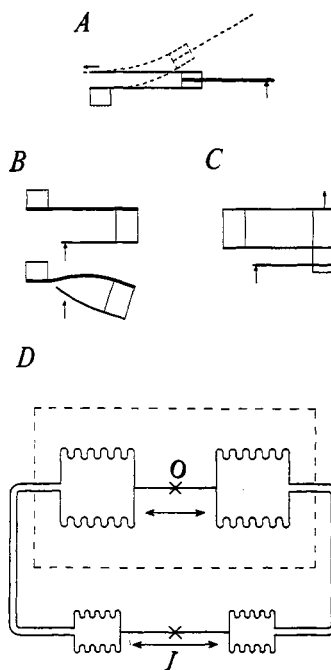


Figure 8. A, twin-strip magnifying or reducing device; B, weak–strong spring system for reducing movement; this does not behave well, since the strong spring suffers a complex distortion; C, weak–strong spring system for reducing movement. Here the strong spring is a parallel spring unit, and so the intermediate block is constrained to remain parallel to its original position; D, pneumatic reducing device. Motion of I causes reduced motion of O.

ends of the strips are connected to a lever or pointer. If the free length of the springs is $a$ and the separation between them is $c$, and the length of the lever from the joined ends of the springs is $b$, the ratio of movement of the end of the lever to that of the moving end of the upper spring is $(a + 2b)/2c$. Convenient magnifications are of the order of 10 to 50, but 400 is feasible. The device can be used the other way round: a transverse force applied to the lever will cause the end of the upper spring to move by the appropriate small amount. This makes a powerful reduction lever which is much more positive in its location of the slowly moving member than the weak–strong spring device described earlier. A variant of the system has been used (Jones 1952) to develop a reduction gear providing the simultaneous forward and reverse movements required for a symmetrically opening spectrometer slit.

Another form of elastic lever system is that based on pneumatic expansion of elastic bellows. de Fonbrune (1950) has made much use of this device in his micromanipulators. Recently, Mr. T. P. Gill at Aberdeen has used the system shown in figure 8D for the making of small adjustments to apparatus in a vacuum enclosure. A movement at I will compress one of the small bellows and expand the other; the consequent change in pressures will cause the larger

197

bellows in the vacuum to move the point O. The reduction will depend on the ratio of the effective cross sections of the bellows. Since the system is balanced, the point B will not be moved, to a first approximation, as air is removed from the vacuum enclosure. The connecting tubes can be flexible polythene, and thus adjustments can be made to a delicate torsion balance in the vacuum, without introducing any vibration.

There are several arrangements dependent on twisted strips. One of the best known is the magnifier (figure 9A)
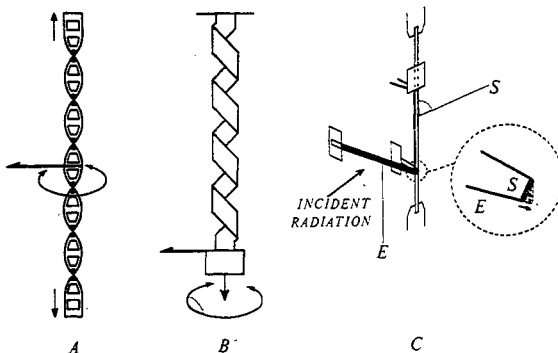


Figure 9. A, Ayrton–Perry twisted strip magnifier as modified by Abramson; B, twisted strip spring used by Wyckoff (1941) in the Gulf gravimeter. A small change in force on the suspended weight causes a large rotation of the lower end of the spring; C, elastic strip system for causing large rotation of the suspension strip S for a change in length of the expansion strip E.

devised by Ayrton and Perry (1884), and modified by Abramson (1933, 1949). In principle, a flat, straight, strip fastened at both ends, is gripped in its middle, and then twisted so as to receive a permanent 'set'; the two halves of the strip thus become twisted in opposite senses. If a pointer is attached to the middle, and one end of the strip is fixed, any small extension of the strip by pulling the other end results in a large angular deflection of the pointer. The National Physical Laboratory (1956) gives formulae for the magnification. The device is very successfully employed in commercial dial gauges which give about 1 mm pointer movement for 1 $\mu$m displacement of the measuring plunger. A few minutes with one of these gauges will give convincing experience of the merit of elastic linkages as regards robustness and absence of friction.

There are other twisted strip devices; one (figure 9B) is coiled strip used in the Gulf gravimeter (Wyckoff 1941); recently this has been used for magnetic susceptibility measurements by Griffiths (1961). Symmetrical arrangements are possible. A further device is the 'serpent-spring' of Tomaschek (1956).

The classic device for magnetic susceptibility measurement is the ring balance devised by Sucksmith (1929), which provides an elegantly simple optical arrangement for measuring small forces.

Another magnifying system (figure 9C) is that used in an infra-red detector (Jones 1959). This is a device for converting translation into rotation, which is effected by stretching a galvanometer-type suspension strip S, and bending a strip E round it, so that this acts as a pair of reins on the suspension strip. The effectiveness of any such device may be assessed by the 'lever arm' given by dividing the applied linear displacement by the angle of rotation. This can be made as small as 50 $\mu$m in the present device, and so a 1 $\mu$m displacement produces about 1° rotation of the suspension.

One satisfactory feature about elastic devices is their extremely small hysteresis. This is illustrated in figure 10, which shows a record of the orientation of the mirror of an infra-red detector using the principle illustrated in figure 9C. The system returns to its zero after deflection to within the limit observable through the thermal drift in the system. This limit is about one scale division, or $5 \times 10^{-11}$ cm, for the system under investigation; it would be very difficult to obtain this performance with anything other than an elastic system.

## 7. Elastic averaging

One of the rather surprising features of Rowland's ruling engine was the fact that he used a lignum vitae nut to transmit the displacement from his leadscrew to the ruling carriage. The use of any wood, even lignum vitae, seems surprising in a precise mechanism, but it may well be that Rowland realized, at least subconsciously, the principle of elastic averaging. This principle has been employed most strikingly by Merton (1950) in his method of making threads devoid of periodic error. A comparatively inaccurate thread inevitably containing periodic errors is cut on an ordinary lathe. A 'nut' consisting of three cork pads spaced at 120° is made to grip this thread; the pads are long enough to grip a thousand or more turns of the thread, and the position of the pad, which to some extent 'gives' to the thrust on each individual turn, is thus determined by the average of many turns, some
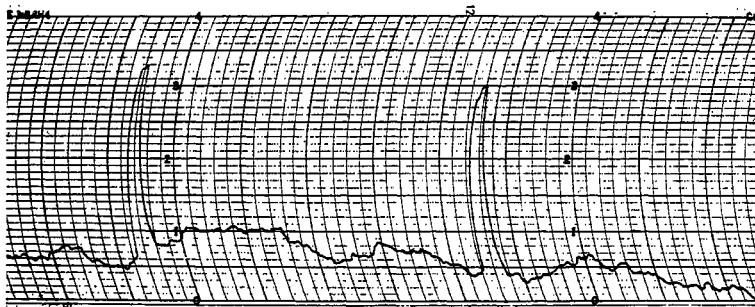


Figure 10. Record of orientation of the mirror in the device of figure 9C. 1 horizontal division = 2 seconds. Sharp deflections are due to incidence of infra-red radiation at a rate of $1 \cdot 7 \times 10^{-9}$ w. One small division represents a movement of about $5 \times 10^{-11}$ cm. The hysteresis appears to be less than this.

of which will be a greater than average pitch, and others smaller. If the Merton nut is used to drive a cutting stylus further along the rod, this stylus will cut a new thread that will be of an order of accuracy $n^{1/2}$ times better than that of the original thread, where $n$ is the number of threads simultaneously contacted by the Merton nut. This enables the original periodic errors to be reduced to a negligible value. We can now see why Rowland used a wooden nut so successfully.

Another application of the same idea was independently tried by Strong (1951) who, to secure improved rectilinearity of motion of his ruling mechanism, mounted several 'ways' in parallel, and made contact with each of these ways with a constraint elastically connected to the moving carriage. The position of the carriage was thus determined by the average of forces exerted by the constraints with the several ways; again the improvement of precision is proportional to the square root of the number of ways.

## 8. Simple harmonic springs

Many springs are required purely to provide a returning force to restore a mechanism to some standard position if displaced. No particular care is required in their construction, but there is scope for ingenuity in their application. For this and many other important aspects of spring technology, a very recent and valuable compendium is *Spring Design and Application*, edited by Chironis (1961).

In physical instruments, a spring is often required to provide a return force that is strictly proportional to the displacement from the unstretched length of the spring. The force then provides a basis for simple harmonic motion, as in a watch or chronometer, or for a regularly calibrated force, as in a spring balance. In these instances, difficulties may arise because of the large temperature coefficients of elasticity of normal materials, which will result in the force constant of the spring changing with temperature. Two possible solutions are open. The first is to compensate by making some other component of the system change its value suitably with temperature. The other is to develop alloys with small temperature coefficients.

The classic instances of compensation were those introduced by Harrison about 1760, and Arnold and Earnshaw between 1780 and 1795, in the development of chronometers; these used the balance spring invented by Hooke about 1660 (Andrade 1950). Harrison controlled the length of his spiral balance spring by a curb which was attached to a bimetallic strip of such a length that the curb moved to shorten the balance spring as the temperature rose (figure 11A). Arnold and Earnshaw accepted the change in force constant of the balance spring, and compensated for it by decreasing the moment of inertia of the balance wheel as the temperature increased. They achieved this by inventing the well-known cut bimetallic wheel that is still standard in watches today (figure 11B and 11C). The device, although very effective, has the disadvantage that the centrifugal force causes the balance weights to fly outwards, thus altering the moment of inertia. This means that the time of oscillation for large swings will differ somewhat from that for small swings, and so the rate of the watch will change slightly as the main spring runs down.

A more recent approach has been to use a low-elasticity coefficient alloy for the balance spring. While this coefficient is usually not zero, it can be small enough to be compensated by a more rigid balance wheel (figure 11D). Here the balance wheel is of one metal, and it is attached to the balance staff

by a single diametrical spoke of another metal, so chosen that the balance becomes oval if the temperature is raised. The consequent small change in moment of inertia is enough to compensate for the change in elasticity of the spring. The amplitude error of the cut balance is thus avoided. The
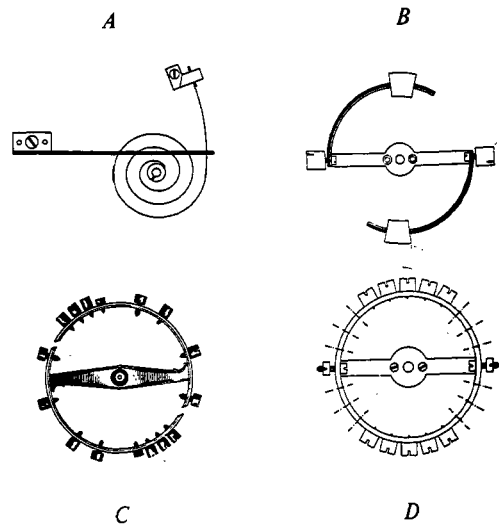


Figure 11. A, Harrison's bimetallic curb for shortening a chronometer balance spring with rise in temperature; B, bimetallic cut balance wheel, decreasing moment of inertia with increasing temperature, invented independently by Arnold and by Earnshaw; C, modern bimetallic cut balance wheel; D, Hamilton uncut balance wheel, with bar and rim of different coefficients of expansion. (*After* R. A. Fell.)

device was invented by Volet in the early 1920's, and subsequently re-invented by Charnwood and by the Hamilton Watch Company. The Company used it in their marine chronometer developed in World War II, and attained with it an accuracy of time keeping such that the error did not exceed 5 msec at any time in a 24 hour run. For details of this and other aspects of the balance and spring see Fell (1955).

## 9. Constant force systems

There are many applications in which a constant force spring, or one having a very small force gradient, would be valuable. The devising of such spring systems has long been a challenge, and several of the attempted solutions have made use of the fact that an ideal straight column buckling under a longitudinal end load exerts a nearly constant restoring force; this fact can sometimes be used if the requirements are not strict.

A variant of the foregoing is the 'Flexator' buckling helical spring (figure 12A), where the increased moments of the buckling forces about the mid-point of the spring can be made to equal the restoring torques exerted by the two halves of the spring, over a range of displacements. The difficulty about such buckling systems is that the component on which they are acting has to be provided with kinematic or elastic constraints, if it has to move along a precise path. This introduces either friction or a positive force gradient; the latter may sometimes be countered by arranging for the buckling spring to have a negative rather than a zero force gradient.

199

In some instances, the positive gradient of an ordinary parallel spring or cross-strip movement may be much reduced by adding a suitably placed weight. With a parallel spring movement where the springs are vertical and the blocks horizontal, with the moving block uppermost, the addition
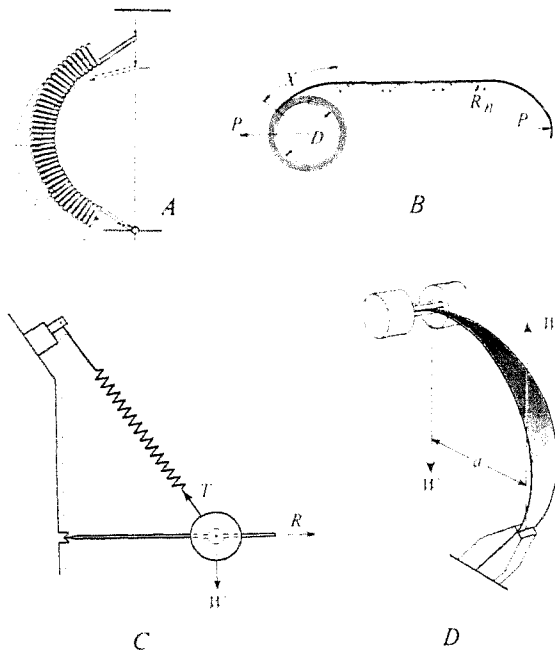


Figure 12. A, 'Flexator' spring (by courtesy of the Hunter Spring Co., Lansdale, Pennsylvania); B, 'Tensator' spring; C, la Coste's vertical seismograph arrangement. The spring is made so as to exert a restoring force proportional to its length, thus automatically closing the triangle of forces $W$, $R$ and $T$; D, elastica, tapered to give constant curvature. If $W$ has suitable magnitude, the increase in bending moment $Wa$, as $W$ moves, very nearly equals the increase in restoring couple exerted by spring, and $W$ is in almost neutral equilibrium.

of weight to the moving block will provide a force tending to bend the springs further when the block is displaced from its equilibrium position. The moment of this force can be made roughly to equal or exceed the restoring moments of the springs. Similarly the restoring torque on a cross-strip hinge operated with its axis of rotation horizontal can be roughly neutralized by adding weight above the axis of rotation, just as the sensitivity of a balance may be increased by a counterweight above the mid-point of the balance beam. A helical spring can sometimes be used instead of a weight for providing the counterforce.

An interesting method of deriving an almost constant force is the 'Tensator' principle (figure 12B) which is described, for example, by Pernetta (1958). In this, a strip is wound into a coil rather like a clock spring, but with the difference that the equilibrium curvature at every point in the spring is the same. The spring thus forms a tight coil, with the inner turns in compression and the outer turns in tension. If the spring is mounted on a mandrel (whose diameter $D$ should be made equal to about $2 \cdot 4 R_n$, where $R_n$ is the natural radius of curvature of the formed spring), and the outer end of the spring attached to a fixed support, then the spring exerts a roughly constant force on the periphery of the cylindrical mandrel as it is unrolled. This is easily seen from energy considerations. As the spring is wound

off the mandrel, and straightened, it stores potential energy in the straightened length. Since the spring in the wound condition has approximately constant curvature, it requires an almost constant amount of work per unit length to change the curvature from this constant value to the zero curvature of a straight line. Thus the force required to extend the spring is also almost constant.

The Tensator spring has the disadvantage that the mandrel must rotate freely, as it is translated, and this usually involves a mounting that entails some friction. It is, however, possible to guide the mandrel with little friction by two stretched threads, parallel with and on either side of the Tensator spring; these can be taken one turn round the mandrel and anchored at the limits of travel. A 4 cm diameter spring will exert a force of some kilogrammes, constant within about 5% over a range of perhaps 20 cm. It thus makes a good substitute for counterweights. Unfortunately, the point to point uniformity is at present not good enough for a truly constant-force device; a spring that is within 5% over 20 cm may well vary by 2 or 3% over 1 cm. Despite this variation, the Tensator spring has an impressive performance and is well worth trying, for example, as a loading device for keeping a kinematically constrained carriage against its driving micrometer. Design data are available from Tensator Ltd., Acton Lane, London, N.W.10.

Another zero or negative stiffness device is the 'sine spring' devised by Wigan (1949). The springs used by Wyckoff (1941) and Tomaschek (1956) have low gradients, and a good deal of development has gone into 'zero length' helical springs, which are so wound that they still exert considerable tension when the turns are in contact. When correctly wound, such a spring can be made to exert a force proportional to its length. This type of spring was used by la Coste (1934) to make a vertical seismograph. This instrument makes most stringent demands on its vertical suspension, which has to neutralize the forces of gravity on its suspended mass, and yet has to have such a small force gradient that the force does not change by more than about 1 part in $10^4$ for 1 cm vertical movement. la Coste's solution was to suspend the mass by a 'zero length' spring, and a boom (figure 12C); it is easy to see that if the force is proportional to the spring length, the triangle of forces is satisfied exactly for any allowed combination of spring length and boom tilt, if the weight has been adjusted for equilibrium in any one position. Thus the system could be ideally neutral, but in practice the temperature coefficient of the spring and creep are limiting factors.

It may be interesting here to mention the elastica, which was investigated by Euler, and which formed, incidentally, the subject of Max Born's doctoral thesis (see Born 1956). If a strip (figure 12D) is weighted enough at one end, and the other end clamped at an angle to the horizontal, a disposition can be found where the moment of the weight about the clamped end just exceeds the elastic restoring torque of the spring. If the weight is less than this value, the spring will flip over to another position of equilibrium. Near the critical point, the force gradient is very low, and periods of oscillation of 20 seconds are possible. The adjustment at this stage is, however, very critical; a change in the initial orientation of the spring of 1 second of arc is sufficient to convert a stable configuration into an unstable one. The device has the unusual property that adding extra mass moves the system to a region where the force gradient is so much stronger that it decreases the period of oscillation. The system is very sensitive to changes of temperature—these can be countered by using an alloy with zero stiffness coeffi-

cient, or by compensating by making the bottom of the spring bimetallic, so as to tend to raise the weight when the temperature rises, and counter the fall caused by the reduced elasticity. This is easy to do by clamping a strip with a different expansion coefficient for a few centimetres on one side of the elastica, and adjusting the clamping point by trial and error. The classical elastica has a varying curvature, but it is possible to make an elastica of constant curvature by varying the width of the strip; this has the advantage of giving constant strain throughout the spring.

While the elastica gives a curved spring, it is possible to devise a pre-curved leaf spring that will have the quasi-neutral property when the load straightens it out. This sometimes allows a more convenient arrangement than can be obtained with a curved elastica.

## 10. Energy stores

Elasticity has been used as an energy store in many applications, such as the longbow and the clock spring. The store is often required to give back its energy at constant force; this was important, for example, in clocks and watches,

then wound on to $D_2$, with reverse curvature to that preformed into it. If both bobbins are free to rotate, the spring will run off the output bobbin on to the storage bobbin, and can be made to exert a substantial torque on the output bobbin. To a first approximation, this torque is constant, as may easily be seen on energy considerations; this is substantially borne out in practice. Figure 13 shows the surprisingly constant torque maintained over 50 turns of the output bobbin. This performance is much better than can be obtained by a conventional spring, and these new springs are being used for long running ciné motors and for recorders in satellites. Design data can be obtained from Tensator Ltd.

## 11. Slow relaxation device

It is sometimes useful to have a mechanical device that will record transient events, but will not respond to slow drifts. A convenient arrangement here is to insert between the recording system and the measuring head a link that may be described as a 'bouncing putty' dashpot (figure 14). The bouncing putty is elastic to sharp displacements and plastic to slow ones; hence the dashpot will follow rapid dis-
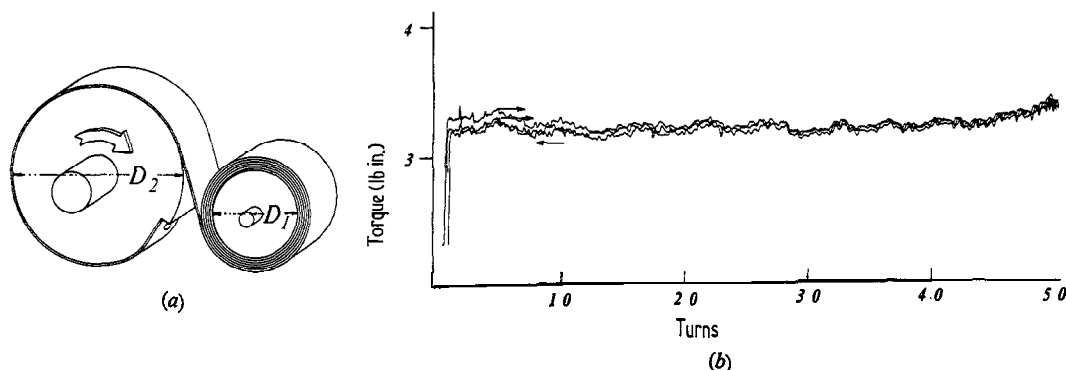


Figure 13. (a) Tensator 'B' type constant-torque motor, with (b) a torque record.
*Reproduced by courtesy of Messrs. Tensator Ltd.*

where the amplitude error referred to above would be brought into play when the balance was given a bigger impulse early in the unwinding of the mainspring than it received in the later stages. For this reason the 'fusee' was invented, which communicated the work from the spring by means of a chain winding on a spiral cam, so that the large force when the spring was fully wound was countered by the small leverage that it could exert on the balance train. Cams are sometimes used for a similar purpose in other spring systems. Harrison not only used the fusee, but he also tapered the thickness of the inner turns of the spring in one of his chronometers, so that the turns maintained themselves clear of one another as the spring was wound, thus reducing the friction. These devices gave a spring system with an output torque that was constant within a few per cent over its entire range.

A new solution has arisen through the development of the Tensator spring, which has resulted in compact constant force energy stores having much better characteristics than ordinary clock springs. In one design, the Tensator spring (which is made in a manner similar to that of the Tensator extension spring described above) is placed on a storage bobbin (figure 13) of diameter $D_1 = 2\cdot4\ R_n$. The outer end of the spring is drawn off the bobbin, and fixed to a second (the 'output') bobbin of diameter $D_2 = 4R_n$. The spring is

placements of the vane immersed in the bouncing putty, but will slowly return to its original position. The time constant can be adjusted to a suitable value between minutes and hours by altering the dimensions of the vane and dashpot or
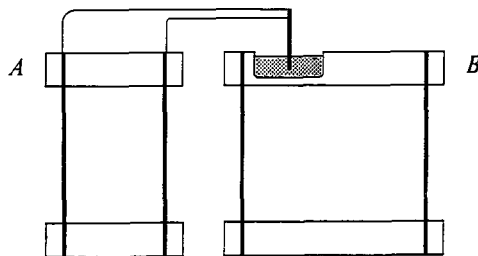


Figure 14. Device for transmitting a rapid displacement, and not responding to a slow one. The displacement is applied to parallel-spring unit A, and transmitted to unit B through a vane embedded in bouncing putty (shown shaded).

the fluidity of the putty, and the recording side will return well (within $10^{-6}$ cm with some putties) to its original position, under the influence of the comparatively weak restoring springs on which it is mounted. We have not yet
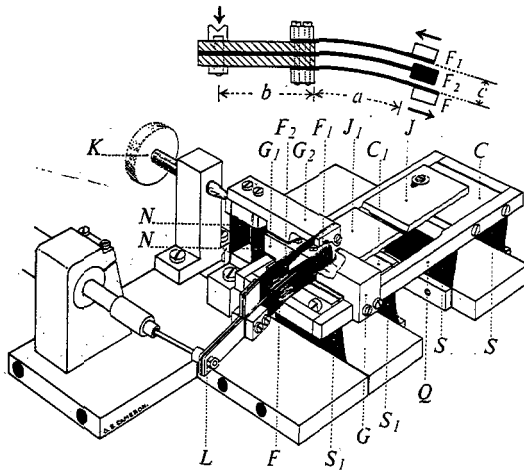
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Figure 15. Symmetrical optical slit, capable of adjustment to within about $0 \cdot 02 \ \mu$m.

### 13. Examples

Apart from the examples of elastic design mentioned in the discourse, there are many to be found in the literature, where the general references (Geary 1954 and 1960, and National Physical Laboratory 1956) will be found useful.

Figures 15 and 16 show two examples with which I have been concerned myself. The former (Jones 1952) was a symmetrical optical slit capable of being controlled in width setting to within about $0 \cdot 02 \ \mu$m; it used parallel-spring units S and $S_1$ for carrying the slit jaws J and $J_1$. The equal and opposite movements for J and $J_1$ were generated by a modified twin-strip mechanism F, $F_1$, and $F_2$, shown in plan at the top of the diagram; figure 16 shows a spring mechanism (Jones 1956a, b) for a symmetrically opening slit with a working length of 10 cm; here the main problem was to arrange for the slit jaws to stay parallel and at constant width as the slit was rotated in a vertical plane. It is worth mentioning a neat device by Crosswhite and Fastie (1956) for simultaneously making the two adjustable curved slits of an Ebert spectrograph by the elastic distortion of a ring.

Various applications of elastic devices used in an experiment involving the precise alinement (better than $10^{-11}$ radian) of an optical system are described in Jones (1961a).
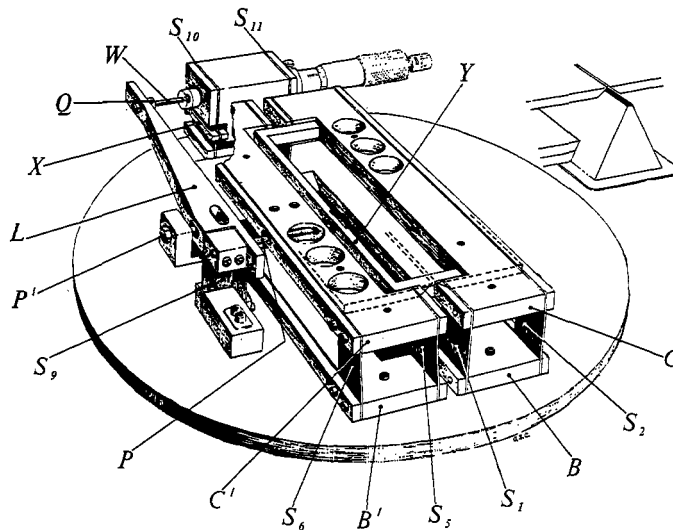


Figure 16. Large symmetrical optical slit mechanism. The jaws are mounted on tables C and C', each of which is carried on parallel-spring systems at both ends. The equal and opposite movements for C and C' are generated by the lever L pivoted on the angle-spring $S_9$, and are transmitted through the pins P and P'.

fully investigated this device for variation with temperature, but it appears to promise one of the few uses for bouncing putty that have so far been found.

### 12. Vibration insulation

In any survey of the uses of elasticity, anti-vibration and anti-shock mountings must be mentioned. The subject is, however, so wide that it is difficult to give anything useful beyond a few key references (Haringx 1950, Elliott and Home Dickson 1959, Snowdon 1958, and Ungar and Pearsons 1961).

### 14. Conclusion

While the value of elasticity in instrument design should not be over-stated, there are some functions in which it gives better or more economically obtained results than does any alternative approach. The design of the mechanical parts of instruments was for a long period after World War I eclipsed by the many advances in electronics. The improvement in sensitivity that these same advances have made possible is now calling for an improved precision of mechanical performance; and this is most easily achieved in some instances,

and notably for small displacements, by the imaginative exploitation of elastic deflection.

## Acknowledgments

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